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A PRE-PUBLICATION REPORT

Investigations in Number, Data, and Space

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Dear Educators:

The TERC *Investigations* authors have almost completed work on an NSF grant-funded revision of the complete K–5 *Investigations* curriculum. The authors intend to keep in place what has been working well and to make selected revisions informed by recent experience and thinking from both research and classroom practice, including *NCTM Principles and Standards for School Mathematics*.

The philosophy of *Investigations* ©2009 remains the same, but the look has changed!

Pearson Scott Foresman is partnering with the TERC *Investigations* authors to publish *Investigations in Number, Data, and Space* ©2009. Here's what you will find:

- Lessons that support how children best learn mathematics
- Teacher-friendly instructional plans that are newly designed and easier to use
- Student Activity Workbooks with daily practice
- Student Math Handbook that summarizes the math and provides game directions
- Content that has been enhanced, extended, and added
- Explicit support of Algebra Connections in the Curriculum Units
- More emphasis on Number and Operations in Grades 3–5
- Greater attention to time and measurement
- Benchmarks to assess mastery at each grade level
- Support for the range of learners in the *Investigations* classroom

Investigations ©2009 will be **greater than** *Investigations* ©2006, ©2004, and ©1998!

The results of a three-year independent longitudinal study (2002–2006) on *Investigations* ©2009 revision materials will follow soon after the release of *Investigations* ©2009.

We have prepared this Pre-Publication Report to give you insight and information about the extent of the revision. **February 2007** is the availability date for *Investigations* ©2009. All grade levels of the revised curriculum will be released together. If you have further questions about the revision, please contact me.

Sincerely,

A handwritten signature in purple ink that reads "Linda Boland".

Linda Boland

Product Manager, *Investigations in Number, Data, and Space*

Pearson Scott Foresman

linda.boland@scottforesman.com

OVERVIEW

OVERVIEW

The Revision of Investigations

Susan Jo Russell, Education Research Collaborative at TERC

The K–5 mathematics curriculum, *Investigations in Number, Data, and Space*, was originally developed from 1990 to 1998 through a grant from the National Science Foundation to TERC. At this writing, we are in the midst of a major revision supported by the National Science Foundation, TERC, and Scott Foresman; this revised version will be available for the school year 2007–2008. While fundamental principles have not changed from one version to the next, the embodiment of the principles has been refined and improved, and the implications of the principles have been revisited, resulting in a deepening of these principles and/or changes in emphasis in our work.

GOALS AND GUIDING PRINCIPLES

From the beginning of our work on this curriculum, 15 years ago, we have had four major goals. These following goals are listed in every curriculum unit:

- Offer students meaningful mathematical problems
- Emphasize depth in mathematical thinking rather than superficial exposure to a series of fragmented topics
- Communicate mathematics content and pedagogy to teachers
- Substantially expand the pool of mathematically literate students

Perhaps even more fundamental to our work are three guiding principles that underlie all our decisions:

1. Students have mathematical ideas.

The curriculum must support all students in developing and expanding those ideas and in making sense of mathematical objects, structures, and connections.

2. Teachers are engaged in ongoing learning about mathematics content and about how students learn mathematics.

Even the best preservice preparation cannot provide enough background in mathematics and student learning. The curriculum must be a professional development tool for teachers.

3. Teachers collaborate with the curriculum materials to create the curriculum as enacted in the classroom.

As we write in the Teacher Note, “Collaborating with the Authors,” which appears in the first unit of each grade level in the current version: “We believe that the only way for a good curriculum to be used well is for teachers to be active participants in continually modifying it. Your role is to observe and listen carefully to your students, to try to understand how they are thinking, and to make decisions, based on these observations”

These are the touchstones we go back to again and again. We approach both students and teachers as agents of their own learning.

ASPECTS OF THE REVISION

1. Computational Fluency

Developing computational fluency with whole numbers must be a lynchpin of the elementary curriculum. This development includes the building blocks of computation: understanding the Base-10 number system and its place value notation, understanding the meaning of the operations and their relationships, knowing the basic addition and multiplication number combinations (the “facts”) and their counterparts for subtraction and division, estimating reasonable results, interpreting problems embedded in contexts, and practicing and consolidating accurate and efficient strategies for computing. It also includes developing curiosity about the characteristics of numbers and operations and learning to articulate, represent, and justify generalizations.

One principle of the *Investigations* curriculum is that time and focus on the building blocks of computational fluency precedes practice and consolidation. Extended time across several grades is spent on each operation. Let’s take subtraction as an example of this process. In kindergarten and Grade 1, students solve subtraction problems by modeling the action of subtraction. By Grade 2, students use the inverse relationship between addition and subtraction to add up to solve problems. During Grades 2 and 3, they become fluent with the subtraction “facts” and model and solve a variety of types of subtraction problems, including comparison and missing-part problems. In Grade 3, they use their understanding of place value to solve problems with larger numbers. In Grades 3 and 4, they articulate, represent, and justify important generalizations about subtraction. For example, if you add the same amount to each number in a subtraction expression, the difference does not change, as in the equation $483 - 197 = 486 - 200$. They analyze and compare strategies for solving subtraction problems. They expand their command of computation procedures with multidigit numbers. At this point, they are also in a position to appreciate the short-cut notation of the U.S. “traditional” or “borrowing” algorithm for subtraction, analyze how it works, and compare it to other algorithms.

The account above gives only a glimpse of the work that helps students develop an understanding of subtraction. Each operation has similar complexity. When we think of entering first graders who are coordinating written and spoken numbers with their quantitative meaning, second graders who are uncovering the relationship between ten tens and one hundred, and fourth graders who are becoming flexible in their use of a number of algorithms to solve, we begin to get some sense of how much work there is to do in these grades.

In the revision of *Investigations*, as we strengthen the coherence and rigor of the number and operations strand, we are determined not to sacrifice the time and depth required for careful development of ideas in this strand. Maintaining a focus on depth and meaning requires us to make difficult decisions in which we navigated the morass of varying state standards, while keeping ourselves grounded in the experience of real students in real classrooms.

To give the attention needed to number and operations, we had to make hard decisions about how much time can be spent on other important mathematical content: geometry, measurement, data, and patterns and

functions. We also considered more carefully how work in these other content areas can connect to and support work in number and operations. For example, a greater emphasis on the foundations of algebra across the grades opens up important opportunities to strengthen work with number and operations. By creating a strong, coherent content strand in patterns and functions across Grades K–5, we were able to connect the work primary students do with repeating patterns to the later work on functions. The work on functions provides interesting problem contexts in which students’ work on ratio and on constant change connect to and support their work on multiplication. Geometry and measurement provide contexts in which students revisit multiplication and fractions. Within the number and operations units, articulation, representation, and justification of general claims about the operations (an aspect of early algebraic thinking) strengthen students’ understanding of the operations.

Making choices about content in the *Investigations* curriculum is based on knowledge from research and practice, including our own extensive field testing. Choice, balance, sequence, and pace of content is based on

- the centrality of number and operations for elementary school students;
- the importance of exposing all elementary students to a range of mathematics content, including work with geometry, measurement, data, patterns, functions, and the foundations of algebra;
- the development of foundational ideas that students need in order to build understanding of mathematics; and
- what we learned from several years in depth work with many diverse students and their teachers.

2. What’s the Math?: Explicitness and Coherence

As *Investigations* has been implemented and we supported professional development and implementation in a variety of ways, we were often surprised to find that what was clear to us about the mathematical focus of a unit, a session, or an activity, and about the connection and sequence of the sessions and activities, was not always so clear to teachers. Sometimes this happened because the meanings of words were not shared. For example, a teacher might say, “But there’s not enough work on place value.” Yet after a conversation, it might become clear that what this teacher meant by “place value” was “expanded notation” rather than a deeper understanding of the Base-10 system. At other times, a teacher might say, “I didn’t understand the point of this activity, so I skipped it,” when we considered the activity crucial. Or a discussion in the curriculum, as implemented, might become a listing of student ideas with little focus or direction. Although “mathematical emphases” are listed at the beginning of every class session, the words used in these emphases did not always help teachers understand the session’s focus.

Considering how to make the purpose, flow, and connections in the curriculum more “explicit” was central to the revision process. Design decisions resulting from these discussions include: 1) more careful choice and wording of the mathematical focus of each investigation and each session; 2) the development of student benchmarks for each unit; 3) clear statement of the focus for each class discussion; 4) a Teacher Note with examples of student work, for each assessment, related directly to the benchmarks for the unit, including analysis of work that meets the benchmark(s), partially meets the benchmark(s), or does not meet the benchmark(s); and

5) greater carefulness about the activities included in each investigation within a unit. Explicitness and coherence help teachers to

- understand the mathematical “story line” of each unit;
- make connections between the mathematical focus of the unit and each activity and assessment; and
- set mathematical goals for each unit and for the school year.

3. Classroom Discussion: Clarity and Focus

In a curriculum that stresses students’ development, investigation, and articulation of mathematical ideas, learning to communicate about those ideas is key. Through talk, as well as through writing, students develop their mathematical reasoning and justify their ideas. Class discussion is one site for this work.

We have become more aware of teachers’ need for help in focusing class discussion so that this precious time can be productive and useful for students. There are often several possible avenues to pursue in a class discussion. The authors must make clear the focus of the discussion, clarify the reason for that discussion in the sequence of mathematical development, provide ways to frame the discussion, suggest questions to ask that are generative and provoke student thinking, suggest ways that students can be prepared to participate in the discussion, define possible outcomes of the discussion, and indicate what ideas are likely *not* to be resolved at the end of the discussion.

In the development of the revised curriculum, we worked to make the purpose and course of each discussion clearer. We provide a clear statement of the discussion focus, initial and follow-up questions that can support student thinking, and a sense of the discussion direction and outcome, including examples of possible student responses. Our revised Dialogue Boxes also have more commentary to help teachers notice particular aspects of the dialogue. Classroom discussion is a critical component of the curriculum and must be supported in the text so that teachers

- focus the discussion on key mathematical ideas;
- ask questions that engage student thinking; and
- know how the discussion connects to the work that precedes and follows it.

4. Support for the Range of Learners

One of the goals that guides our curriculum design process is to “expand the pool of mathematically literate students.” This goal requires us to reflect on whether the curriculum engages all students in significant mathematical work and how to support teachers in making modifications to meet their needs. In the revision, we have designed two new elements that focus on this goal.

First, throughout the units, many of the activities include a section called “Supporting the Range of Learners.” These sections offer suggestions gleaned from classroom experience about how to modify and/or extend activities for the variety of students in the class while maintaining focus on the important mathematical ideas. Second, a set of cases at each grade level, written by teachers and framed with commentary written by staff, provide examples of how teachers think through the issues of diversity in their classrooms. These cases are

not meant to be exemplars nor can they provide ready-made solutions for other classrooms; rather, they show the ways in which teachers consider how the needs and strengths of their students vary, how they pose questions about their own teaching, and how they examine the results of different teaching strategies. The cases are organized into the following categories: 1) Developing the math community; 2) modifications; and 3) language and discussion. These supports in the curriculum share thinking, based on ideas from teachers in diverse classrooms, to help users of the curriculum

- facilitate the active participation of all students in mathematics;
- acknowledge and build on the strengths students bring to the classroom from their own communities, cultures, and language;
- respect the intellectual work of all students, including those who are struggling; and
- develop modifications when students are having difficulty or need more challenge.

The work on which this essay is based was funded in part by the National Science Foundation through Grant No. ESI-0095450 to TERC. Any opinions, findings, conclusions, or recommendations expressed here are those of the author and do not necessarily reflect the views of the National Science Foundation.

OVERVIEW

Revision Project Collaborators

A key part of the revision process involves drawing on the expertise and experience of a diverse group of teachers, teacher educators, administrators, and mathematicians. This group includes both teachers and administrators who have been using the *Investigations* curriculum in their classrooms and school systems for many years as well as those new to the curriculum. They bring us invaluable experience from their work facing the complex issues of using curriculum as a tool to improve mathematics learning and teaching.

TEACHER COLLABORATORS

This group of 67 teachers from 26 different school districts was formed through application from local schools and through administrator recommendations in nonlocal schools. They represent schools in large urban systems, small urban centers, suburban communities, and small rural towns. The groups include teachers of varied ethnic and language backgrounds, those who teach in bilingual classrooms and in classrooms with significant numbers of special-needs students. These “front line” teachers work with the TERC staff to develop the revised materials. They may try out new activities, complete units, and/or a whole year of the revised curriculum in their classrooms, providing feedback to staff. Feedback is given in the form of written comments, extensive classroom observations, videotaping of classrooms, and meetings with staff. Many of these teachers have also written classroom episodes that are being used directly in the revised curriculum in a number of ways, for example, as the basis for Dialogue Boxes or for a new feature of the curriculum—snapshots of classroom moments that focus on working with the range of learners in the classroom.

INVESTIGATIONS WORKSHOP LEADERS

This established group of more than 40 educators from across the country staff the *Investigations* summer workshops. About half of the group are classroom teachers who actually teach the curriculum, and many of them have additional leadership responsibilities in their home districts. Others are in school district professional development positions or work as educational consultants. People of color and fluent Spanish speakers make up about 35% of the group. This group is very experienced both in using *Investigations* with a wide range of students and in providing professional development for other teachers using the curriculum. They provide input to the Revision staff and advice about professional development to support the revised materials.

ADVISORS AND CONSULTANTS

The following provide advice to staff on a variety of topics, including mathematical content, pedagogy, assessment, and supporting the range of learners. They include teachers, administrators, teacher educators, and mathematicians.

Deborah Ball

William H. Payne Collegiate Professor in Education
University of Michigan

Hyman Bass

Roger Lyndon Collegiate Professor of Mathematics and Mathematics Education
University of Michigan

Maria Blanton

Associate Professor
Department of Mathematics
University of Massachusetts Dartmouth

Mary Canner

Principal
Natick (MA) Public Schools

Thomas Carpenter

Professor of Curriculum and Instruction
Wisconsin Center for Education Research

Ronit Eliav

Teacher
New York City Public Schools

Michael Flynn

Teacher
Southampton (MA) Public Schools

Janis Freckman

Elementary Mathematics Coordinator
Milwaukee Public Schools

Lynne Godfrey

Mathematics Coach
Cambridge (MA) Public Schools

Ginger Hanlon

Instructional Specialist Mathematics, Region 9
New York City Public Schools

DeAnn Huinker

Director, Center for Mathematics and Science Education Research
University of Wisconsin Milwaukee

Jim Kaput

Professor of Mathematics
University of Massachusetts Dartmouth

Kate Kline

Associate Professor
Department of Mathematics and Statistics
Western Michigan University

Jim Lewis

Professor
Department of Mathematics and Statistics
University of Nebraska Lincoln

Laurie Link

K-5 Mathematics Curriculum Specialist
Concord (MA) Public Schools

William McCallum

Professor of Mathematics
University of Arizona

Steve Monk

Professor of Mathematics
University of Washington

Harriet S. Pollatsek

Professor of Mathematics
Mount Holyoke College

Debbie Shein-Gerson

Elementary Mathematics Specialist
Weston Public Schools

Gary Shevell

Assistant Principal
New York City Public Schools

Liz Sweeney

Elementary Math Department
Boston Public Schools

Ana Vaisenstein

Teacher
Boston Public Schools

Lucy West

Consultant
Metamorphosis TLC Inc.

Lois A. Womack

Mathematics Teaching Specialist
Milwaukee Public Schools

AUTHORSHIP TEAM

AUTHORSHIP TEAM

Author Biographies

VIRGINIA BASTABLE

Virginia Bastable is the Director of the SummerMath for Teachers Program at Mount Holyoke College and a senior writer on the Grades 3–5 *Revisions of Investigations in Number, Data, and Space*® curriculum project with a special focus on early algebra. Dr. Bastable is also a principal investigator and developer of the *Developing Mathematical Ideas* Professional Development Curriculum. She has also coauthored several papers about early algebra, including *Classroom Stories: Examples of Elementary Students Engaged in Early Algebra* and *Early Algebra: What Does Understanding the Laws of Arithmetic Mean in the Elementary Grades?* Prior to directing SummerMath, Virginia was a secondary school classroom teacher.

KATIE BLOOMFIELD

Katie Bloomfield is a Research and Development Specialist at the Education Research Collaborative at TERC in Cambridge, MA and a senior writer on the Grades 3–5 *Revisions of Investigations in Number, Data, and Space*® curriculum project. Katie is a former third and fourth grade teacher, and she was a field test teacher for the development of *Investigations*. As a member of the Investigations Implementation Center and the Investigations Workshop project, Katie designed and taught professional development workshops nationally for teachers and school districts implementing *Investigations*.

KEITH COCHRAN

Keith Cochran is a Research and Development Specialist at the Education Research Collaborative at TERC in Cambridge, MA and a senior writer on the Grades 3–5 *Revisions of Investigations in Number, Data, and Space*® curriculum project. His background is in classroom teaching and professional development in mathematics for teachers and administrators. His work in the Clark County School District (Las Vegas, NV) included the initial implementation of *Investigations* and serving as the district site leader for the field tests of *Bridges to the Mathematics Classroom*, *Developing Mathematical Ideas*, and *Lenses on Learning*. He has served on the staff of the DMI Leadership Institutes and is the mathematics resource partner for Bank Street College's *Teachers for a New Era* initiative.

KAREN ECONOMOPOULOS

Karen Economopoulos is a project director at the Education Research Collaborative at TERC in Cambridge, MA. She is the codirector of the *Revisions of Investigations in Number, Data, and Space*® curriculum project and the director of the K–2 development team. Prior to this, she codirected the *Investigations* Implementation Center. As such, she worked extensively with classroom teachers, administrators, and school districts in the area of curriculum reform. A former K–2 classroom teacher, Karen directed the K–2 development of *Investigations in Number, Data, and Space*®. She coauthored *Beyond Arithmetic: Changing Mathematics in the Elementary Classroom* and directed an Exxon-funded staff development project focused on mathematics in the kindergarten classroom. Karen has served on the Mathematical Sciences Education Board and is a former consultant with Marilyn Burns Education Associates.

NANCY HOROWITZ

Nancy Horowitz is a Research and Development Specialist at the Education Research Collaborative at TERC in Cambridge, MA and a senior writer on the Grades 3–5 *Revisions of Investigations in Number, Data, and Space*® curriculum project. Prior to her work as a curriculum developer, Nancy was a classroom teacher and mathematics coach. She is also a certified teacher of English as a Second Language and has worked extensively with English language learners. Nancy was a field test teacher for the development of *Investigations* and a participant in several NSF-funded teacher development projects, including *Teaching to the Big Ideas* and the *Teacher Curriculum Partnership project*. Nancy has designed and taught professional development workshops nationally for teachers and school districts implementing *Investigations*.

MEGAN MURRAY

Megan Murray is a Research and Development Specialist at the Education Research Collaborative at TERC in Cambridge, MA and a senior writer on the Grades K–2 *Revisions of Investigations in Number, Data, and Space*® curriculum project. Megan is also a staff member of the *Investigations* Implementation Center, which supports teachers, schools, and districts that are implementing the *Investigations* curriculum. Megan has been involved in many aspects of the *Investigations* curriculum, including the initial field test and development. Megan is the author of *Schools and Families: Creating a Math Partnership*, an extensive guide aimed at helping teachers and administrators answer the questions, “How do we engage parents as partners in the work of improving mathematics teaching and learning for all students?”

SUSAN JO RUSSELL

Susan Jo Russell is a principal scientist at the Education Research Collaborative at TERC in Cambridge, MA. She is the codirector of the *Revisions of Investigations in Number, Data, and Space*® curriculum project and was the principal investigator of the original *Investigations* curriculum as well as *Developing Mathematical Ideas*, a professional development curriculum series for elementary and middle-grade teachers. Dr. Russell has worked at TERC for more than 20 years, directing projects that have focused on computer education, mathematics for special-needs students, professional development in mathematics for elementary and middle school teachers, research on students’ and teachers’ understanding of mathematics, and curriculum design for elementary students. Her current work focuses on issues of professional development and leadership development in elementary mathematics and on understanding how practicing teachers can learn more about mathematics and about children’s mathematical thinking.

DEBORAH SCHIFTER

Deborah Schifter is a senior scientist at the Education Development Center in Newton, MA and a senior writer on the Grades K–2 *Revisions of Investigations in Number, Data, and Space*® curriculum project with a special focus on early algebra. Dr. Schifter has worked as an applied mathematician, has taught elementary, secondary, and college level mathematics and, since 1985, has been a mathematics teacher educator. She is a principal investigator and developer of *Developing Mathematical Ideas*, a professional development curriculum series for elementary and middle-grade teachers. Her team is currently working on the final two modules: *Reasoning Algebraically about Operations and Patterns, Functions, and Change*.

LUCY WITTENBERG

Lucy Wittenberg is a project director at the Education Research Collaborative at TERC in Cambridge, MA. She is the director of the Grades 3–5 development team of the *Revisions of Investigations in Number, Data, and Space*® curriculum project. Prior to this work, Lucy was a member of the *Investigations* Implementation Center and *Investigations* Workshops project. She has extensive experience supporting teachers and administrators with the implementation of the *Investigations* curriculum. She is also a contributing author of professional development curriculum, *Bridges to Classroom Mathematics Project*. Lucy is a former third and fourth grade teacher.

CONTRIBUTING AUTHORS

Arusha Hollister

Erin Leidl

Young Oh

Darrell Earnest

Stephen Monk

Jennifer diBrienza

Denise Bauman

Cornelia Tierney

Jan Mokros

Student Math Handbook

Beth Perry

Kathy Sillman

Range of Learners

Hollee Freeman

Mary Beth O'Connor

Judy Storeygard

Ana Vaisenstein

Technology

Jim Hammerman

Classroom Field Work

Amy Appell

Rachel Davis

Traci Higgins

Julia Thompson

ADMINISTRATIVE STAFF

Amy Taber, Project Manager

Beth Bergeron

RESEARCH

RESEARCH

Reflections on ©2009

Field-test Teachers and a Math Advisory Board Member

“When I was asked to field test the new units for the *Investigations* revisions project, I didn’t know what to expect. I really liked the original units and had finally gotten to a point where I felt I knew them pretty well. I wasn’t sure if I wanted to relearn a new curriculum (especially while being observed intensely by the field-test observers).

However, part of me was also very curious. What would the new units be like? How much would the authors change? Would they keep some of my favorite sessions? Most of all, I wondered whether the units would have the same feel as the old ones. I enjoyed having a well thought-out curriculum that built on prior knowledge and helped kids develop a deep understanding of math concepts.

I’m glad I let my curiosity win over my initial trepidation because the new units are incredible. First of all, they are similar enough to the older units so I didn’t have to start from scratch. I already knew the flow of a typical *Investigations* unit. Yet I felt there was a clearer vision of where my students needed to go in terms of the mathematical understanding.

Specifically, there is a strong emphasis on students developing computational fluency. The same goal was in the original units, but there is much more emphasis on it with the new materials. Sessions and Math Workshop (formerly Choice Time) activities are geared toward students’ development of efficient and effective strategies for solving problems. Students work toward mastering their addition facts and develop an understanding of the Base-10 structure of our number system.

In addition, the new materials engage students in all aspects of mathematics (number, geometry, data, patterns, functions, and measurement). I am happy to say that the authors stayed true to their original vision for a thorough and deep approach to teaching math. Teachers will not be disappointed with the revised units.”

Michael Flynn, TERC Investigations ©2009 Field-Test Teacher

“As a second grade teacher, I am very excited about the revised *Investigations* materials. Developing an understanding of place value is a big piece of the work that should be done in second grade. Understanding place value means understanding how our Base-10 number system works and understanding the role that “10” plays in that number system. They need to explore how using 10 and chunks of 10 can make their computation more efficient.

The new materials address these issues explicitly and do so using contexts that are meaningful and engaging to the children. My second graders love using the idea of a sticker store to explore these ideas. The new material was especially helpful to children who were struggling with these ideas.”

Karen Schweitzer, TERC Investigations ©2009 Field-Test Teacher

“I really enjoyed being a part of the revisions process. As a long-term user of the *Investigations* curriculum, it was very interesting to see and participate in the changes that were being made to the curriculum. The new work, which focused the children so much more explicitly on their strategies, enabled my students to become much more mathematical in their thinking. I even joked with the authors that they had turned my students into “mathematical monsters” since the students now had more ways to solve problems that were more solid and more efficient. The revised work enabled struggling students to more easily become much stronger in their mathematical understanding. As part of the revisions work, I also wrote about what was happening in my classroom. As I wrote about my students’ thinking, I became better at understanding the mathematics and what my students were grappling with as they developed their individual mathematical thinking.”

Susan Smith, *TERC Investigations* ©2009 Field-Test Teacher

“I was lucky to be one of the classroom teachers who field-tested the revised TERC units for three years. When I was first asked to be a part of this project, I didn’t know what to expect. I certainly didn’t expect to be doing math as an adult. At the time, I had no idea how vital deepening my understanding of mathematics was in my development as a teacher of mathematics. The depth of my understandings has significantly impacted my ability to develop the math skills of my third graders.

One strength of the TERC revised units is that they include basic skills through, and including, higher-level thinking skills. An example of this is in the Grade 3 Patterns and Functions unit. Students are learning about changes over time, specifically reading temperature graphs from places around the world. In this unit, while students are learning to read these graphs, they have to learn what to do with the black line that’s halfway between 40 and 50 degrees. They have practice plotting temperatures on these graphs, learning how and where to plot these points. The unit extends their thinking through discussing the temperature changes over a period of months. Students are asked to describe what kinds of changes happen and use mathematical language to describe these changes whether they are describing constant or steep slopes.

The *Investigations* units help students learn basic skills and higher-level thinking math skills not only in the nonnumber units, but also in the number units. The new units provide explicit support for teachers to be sure that all students experience these basic skills and higher-level thinking skills. Oftentimes in a particular program, whether it be literacy or math, you may find a heavy push for basic skills or higher-level thinking skills, but it is rare to find a program that includes both and makes it clear to teachers and students that both are a priority for all learners. TERC does this successfully, weaving these skills through all of their units and highlighting for teachers how to involve all students in this learning.”

Shoshanah Starr, *TERC Investigations* ©2009 Field-Test Teacher

“Participating in the revisions project as a field-test teacher was the best professional development experience I had in years! I learned to pay closer attention to algebraic ideas my first grade students discovered through games. For example, when playing the On and Off game, or Counters in a Cup, my students usually noticed that the quantities would flip-flop: on one turn, there were 4 counters on and 3 counters off and, on another turn, there were 3 counters on and 4 counters off. Students always showed a fascination with those reversals. However, during revisions, I stayed with these preliminary observations much longer than I used to because I knew my students were noticing the commutative property of addition. My end goal was for them to become efficient problem solvers. If they could clearly articulate that $3 + 4 = 4 + 3$ and that it was true with any numbers in any situation, then they could use this property to solve any problem.

Throughout the year, we had many discussions that revealed that this idea was not obvious for my students. For example, they didn't know whether “the turn-arounds” were true for any numbers in any situation. We had lots of passionate discussions, which helped them generalize this idea and solve $3 + 15$ by reversing it to $15 + 3$, not because I had told them to do so, but because they understood that the order of the addends doesn't affect the total. And all this understanding took place in first grade!”

Ana Vasenstein, *TERC Investigations* ©2009 Field-Test Teacher

“I enjoyed discovering how much mathematics goes on in the elementary classroom. The vignettes we looked at teemed with mathematical life, showing students grappling to make real and solid their fragmentary notions about the inner workings of the number system. I also enjoyed giving advice, of course. The right balance between respect for the mathematics and attention to student thinking is hard to find. I saw it as my job to ensure attention to the former, while enjoying the pleasant side benefit of learning more about the latter. Observing the engagement of teachers, educators, and administrators in the *Investigations* development process, I was struck by the relative poverty of opportunities for mathematicians in K–12 education. I hope that my work, and the work of the other mathematicians on the Advisory Board, is part of a more constructive and influential role for my profession.”

Bill McCallum, University of Arizona, *TERC Investigations* ©2009 Math Advisory Board

RESEARCH

Development and Evaluation

The 5-year development process of this revision includes: 1) gathering and analyzing input for the revision; 2) development and formative evaluation (activity testing and unit testing); and 3) full-year field testing and final revision for publication. Activity testing, unit testing, and field testing involved 67 field-test teachers representing 26 school districts.

	Year 1	Year 2	Year 3	Year 4	Year 5
Data gathering	X				
Activity testing	X	X			
Unit testing		X			
Full-year field testing		X	X		
Final revision, editing, design			X	X	X

INITIAL DATA GATHERING

In Year 1, TERC authors gathered data about the activities and units that have worked well and those that have been problematic (and why); the revisions and adaptations that teachers or school systems have made (and why); the logistical, practical, and/or sequential aspects that need to be improved; and professional development needs.

DESIGN AND FORMATIVE EVALUATION: ACTIVITY TESTING AND UNIT TESTING

Activity testing involves the design and trial of new activities—usually a series of connected lessons. The activity tests are carried out by the classroom teacher, alone or with the developer, with extensive field notes taken by an observer. Activity testing is carried out for new or significantly revised activities. Unit testing takes place when a complete curriculum unit is ready in draft form. The unit, 3 to 6 weeks in length, is tested in 3 to 5 classrooms. Classrooms are observed once or twice per week; student work is collected; and teachers are interviewed several times, during and after the unit trial.

FULL-YEAR FIELD TESTING AND REVISION

Year-long grade-level testing is carried out for each complete grade level at local sites that can be observed by staff and also at several distant sites. During year-long testing, data is gathered from local sites as follows: carefully documented classroom observations; periodic meetings with teachers (individually, in grade levels, and as a whole K–5 group); selected student work (including assessments); notes written on units by field-test teachers. The authors gather data from distant sites as follows: selected student work (including assessments), written reports from teachers at the conclusion of each curriculum unit; staff visits to observe classrooms; meetings with site coordinators. Revision of the curriculum is ongoing as the grade-level testing proceeds.

NOTE: Summative evaluation, in the form of a longitudinal study carried out by a team of researchers at Indiana University, is summarized in Update: What Did They Learn?

RESEARCH

Algebra in the Revision

Susan Jo Russell, Deborah Schifter, and Virginia Bastable

A major emphasis of the revision is to make the foundations of algebra currently in the curriculum more visible to teachers and students, to expand this work in the context of students' study of number and operations, and to deepen the focus on patterns, functions, and change by creating a consistent strand in Grades K–5.

Algebra is a multifaceted area of mathematics content, and various schema have been proposed for classifying that content (e.g., Kaput, 1998, 1999; Usiskin, 1988). These references, as well as the NCTM's Principles and Standards, suggest several central aspects of algebra (these are related, overlapping categories): a) generalizing and formalizing patterns; b) representing and analyzing the structure of number and operations; c) using symbolic notation to express functions and relations; d) representing and analyzing change. In the *Investigations* revision, we have addressed these four aspects in two major ways.

Integration of Early Algebra into the Units That Focus on Number and Operations

The materials highlight the generalizations about number and operations students frequently observe. Teachers learn to help students articulate these generalizations and challenge them to consider the questions: Does this generalization apply to all numbers (in the domain under consideration)? Why does it work? How do you know? Throughout Grades 1–5, students articulate, represent, investigate, and justify general claims. In each of the number and operations units, an essay, *Algebra Connections in This Unit*, highlights several generalizations and includes examples of how students think about and represent them. Investigation and discussion of some of these generalizations are built into unit sessions; at other times, “Algebra Notes” alert the teacher to sessions in which these ideas are likely to arise. For example, in Grade 2, students consider whether the order of terms affects the sum in addition problems or the difference in subtraction. In Grade 3, students discuss the generalization underlying the equivalence of subtraction expressions as in the equation, $104 - 78 = 106 - 80$. In Grade 5, students justify why halving one factor and doubling the other in multiplication results in the same product (e.g., $65 \times 24 = 130 \times 12 = 1560$).

A Complete K–5 Strand on Patterns, Functions, and Change

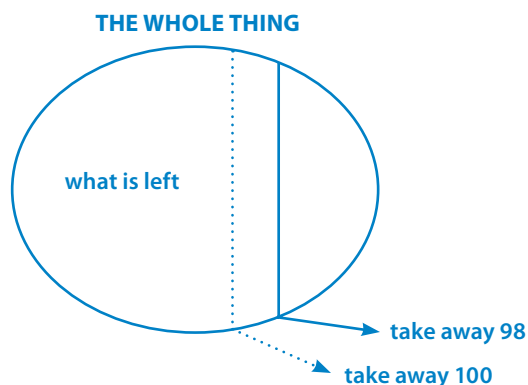
We have created a coherent K–5 strand that starts with repeating patterns and number sequences in Grades K and 1, connects to functional relationships beginning in Grade 2, and focuses on linear and nonlinear change in Grades 3–5. Students study relationships that follow rules (such as the relationship between the number of windows in a building and the number of floors if the building has a fixed number of windows per floor) and relationships that do not follow rules (such as the relationship between temperature and time in Grade 3 and between plant growth and time in Grade 4). They work extensively with ways of representing these relationships: in words, with numbers, with tables and graphs, and (starting in Grade 4) with symbolic notation. These units reinforce and connect with work in other units on multiplication, ratio, area, volume, and graphing.

EXAMPLES FROM THE CLASSROOM

To provide a flavor of the kind of work students do in the revised curriculum, here are two examples from field-test classrooms that fall under the first category: making and justifying general claims about number and operations.

Grade 4: Is It 2 More or 2 Less?

In a Grade 4 class, students were trying to explain why the result of the subtraction, $145 - 98$, is two more, not two less, than the result of the subtraction, $145 - 100$. The context for the problem was related to a science project in which the class weighed apples in grams as they dried out. To address the subtraction problem, Brian drew a closed shape representing an “apple” divided into two parts, then used it to show what would happen if the part that is “taken away” is diminished while the whole stays the same. “See, this is the apple at first,” he explained. “And you take some away [the part to the right of the dotted line] and have some left [to the left of the dotted line]. Then you take away 98 grams instead, so it’s over here [the part to the right of the solid line is now the part that is subtracted; left of the solid line is what remains].”



With the presence of this picture to focus the discussion, more students joined in, using the representation not only to reason about the particular numbers, but to state and justify a more general claim. Rebecca said, “It’s like you have this big hunk of bread and you can take a tiny bite or a bigger bite. If you take away smaller, you end up with bigger.” Then Max stated: “The less you subtract, the more you end up with, AND in fact the thing you end up with is exactly as much larger as the amount less that you subtracted.”

Grade 2: Switching Around the Numbers

But what about students in the primary grades? Aren’t they “concrete” thinkers? In fact, young students, too, notice regularities about the work they do as they count, compare quantities, and learn about addition and subtraction. Here is an example from a second grade. The teacher asked the students to generate combinations of two addends with a sum of 25. As they listed these on chart paper, students soon noticed—as they had before in their computation work—that they could “switch around” the numbers and still get the same sum, for example,

if $23 + 2 = 25$, then $2 + 23 = 25$. In fact, the teacher had in mind that this idea would come up during this activity and had planned follow-up questions. She asked, “Suppose I asked . . . if you could prove that or explain it better to me . . . if we take the 2 and put it first, do we still get 25?” Nikki demonstrated with a stack of 23 cubes and a stack of 2 cubes. She moved the 2-cube stack rapidly and repeatedly from the right side to the left side of the 23-cube stack. “It doesn’t matter,” she said, “if you keep on just switching it around, it will still make 25 . . . you’re not taking away or adding to it . . . it will still be the same number.” Again, in this example, the use of a representation that embodies the operation enables the students to reason about the general claim. Although Nikki is holding particular quantities—23 and 2—her reasoning applies to any pair of numbers. Once all the students seemed quite convinced that the order of any pair of numbers in an addition expression could be changed without changing the sum, the teacher asked the students if the same is true for subtraction. From her experience with these ideas, the teacher knew not to assume that students thought that the “switch around” rule applies only to addition. Students thought about her question for a few minutes, then several students offered their ideas, using $7 - 3$ and $3 - 7$ as an example.

Nikki: If you have 3 take away 7, but 3 doesn’t have 7. . . . You can only take away 3 to make zero.

Alita: You can’t use the 3 because after you use the 3—3, 2, 1, 0, 0, 0. . . the zero’s going to keep on repeating itself.

Edward: It wouldn’t be zero. It would be negative 4. . . . That means you’re going lower. If you’re going lower than zero, that means negative 1, negative 2, negative 3. . . .

Although these students did not yet have all the number experience necessary to understand this idea, the teacher noticed that they were making important observations about the differences between the properties of addition and the properties of subtraction. She planned to return to this discussion as other opportunities arose—for example, can the order of the numbers be changed in an addition expression with more than two addends without affecting the sum?

ALGEBRA FOR ALL STUDENTS

The work of generalizing and justifying in the elementary classroom has the potential of enhancing the learning of all students. The teachers with whom we have collaborated for several years have realized this potential in their classrooms. Teacher collaborators report to us that students who tend to have difficulty in mathematics become stronger mathematical thinkers through this work. As one teacher wrote, “When I began to work on generalizations with my students, I noticed a shift in my less capable learners. Things seemed more accessible to them.” When the generalizations are made explicit—through language and through spatial representations used to justify them—they become accessible to more students and can become the foundation for greater computational fluency. Furthermore, the disposition to create a representation when a mathematical question arises supports students in reasoning through their confusions. Brian (in the Grade 4 example above), a tentative learner in mathematics, created a representation that illuminated an important idea. In the second

grade classroom, in an urban center with a historically large proportion of underachieving students, a range of students offered important ideas about how addition is and subtraction is not commutative.

At the same time, students who generally outperform their peers in mathematics find this content challenging and stimulating. The study of number and operations extends beyond efficient computation to the excitement of making and proving conjectures about mathematical relationships that apply to an infinite class of numbers. A teacher explained, “Students develop a habit of mind of looking beyond the activity to search for something more, some broader mathematical context to fit the experience into.” In the fourth grade example above, Max, one of the most mathematically successful students in the class, listened carefully to his classmates’ explanations and then enjoyed the challenge of formulating a precise statement of the generalization. And Edward (in the Grade 2 example), who knew more about numbers than his peers, was able to seed the conversation with a new idea about numbers below zero.

EARLY ALGEBRA IS FUNDAMENTAL

Underlying these kinds of discussions are what one of our mathematician advisors calls “foundational principles”—principles that connect elementary students’ work in arithmetic to later work in algebra. For example, the idea explored by the fourth graders (the less you subtract, the more you have left) can be represented as, “If $a - b = c$, then $a - (b - x) = c + x$,” or, more concisely, “ $a - (b - x) = (a - b) + x$.” A discussion among middle schoolers similar to that in the 4th grade example could provide an opportunity to consider why the associative property does not apply to subtraction, and to articulate a rule that does. The second graders do not yet have the experience with negative numbers to allow them to completely make sense of $3 - 7$, but they are nevertheless engaged in reasoning about foundational ideas, in this case, that addition is commutative, but subtraction is not: $a + b = b + a$, but $c - d \neq d - c$. In later years, they will come to see that there is a regularity here, that if $c - d = a$, then $d - c = -a$, or $c - d = -(d - c)$.

For most adults, notation such as the use of variables, operations, and equal signs is the chief identifying feature of algebra. Although we do introduce symbolic notation in Grade 4, the notation is not the focus of activity in Grades K–5. Underlying the notation are ways of reasoning about how the operations work. This reasoning—about how numbers can be put together and taken apart under different operations or about relationships between two changing quantities—not the notation, is the central work of elementary students in algebra.

In the course of our work to integrate the foundations of algebra into the *Investigations* curriculum and through the insights of our field-test teachers and the thinking of their students, we have concluded that work in early algebra is fundamental to the experience of young students. In summary:

1. Early algebra is not an add-on. The foundations of algebra arise naturally throughout students’ work on number, operations, patterns, and through noticing how one thing changes in relation to another. This work anchors students’ concepts of the operations and underlies greater computational flexibility.

2. Algebra as generalized arithmetic provides openings for working on reasoning and proving.
3. Algebra provides the opportunity to learn about the power of representation as a basis for mathematical reasoning.
4. Working on the algebraic underpinnings of arithmetic is one way of engaging the range of learners in mathematical thinking.

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Computational Algorithms and Procedures

Learning, with understanding, about computation and algorithms to solve problems is a natural and fertile site for learning about the detailed nature of numbers and operations and their various models and representations. [Hyman Bass, “Computational Fluency, Algorithms, and Mathematical Proficiency: One Mathematician’s Perspective,” Teaching Children Mathematics, Feb. 2003, p. 326].

In the revised version, *Investigations* will continue to have a major emphasis on developing computational fluency for whole-number operations through work that supports students to understand the meaning of the operations and the structure of the Base-10 number system. This work includes becoming fluent with “basic facts,” developing efficient, accurate, and flexible methods for solving computation problems, making and justifying generalizations about the operations, and analyzing and comparing a variety of algorithms.

Although each student may settle on one primary method for solving problems in each operation, students are expected to study more than one algorithm for each operation. Students study a variety of algorithms for a number of reasons, including:

- Justifying different algorithms or procedures provides insight into characteristics of the operations. Comparing algorithms also illuminates underlying properties.
- Access to different algorithms and procedures leads to flexibility in solving problems. One method may be better suited to a particular problem; one method can be used to check another.

In students’ study of operations in the *Investigations* curriculum, there is a time when they build strategies they are comfortable with, that make sense to them, and that they can gradually apply to harder problems. They use what have been called more “transparent” algorithms and methods, in which the properties of the operations and the place value of the numbers are not hidden by shortcut notation.

An algorithm that will be used to program a machine must be efficient to achieve computational speed, but does not have to be transparent. If humans will learn and use the algorithm, however, transparency and ease of use are important [Ibid., p. 324].

For example, in these two methods of solving a multiplication problem, the values of all the partial products and the application of the distributive property can be seen clearly.

Method 1

$$\begin{array}{r} 59 \\ \times 72 \\ \hline 18 \\ 100 \\ 630 \\ \hline 3500 \\ 4248 \end{array}$$

Method 2

$$\begin{array}{r} 59 \times 72 = (60 \times 72) - (1 \times 72) \\ 4200 \quad 60 \times 70 \\ +120 \quad 60 \times 2 \\ \hline 4320 \\ -72 \quad 1 \times 72 \\ \hline 4248 \end{array}$$

Later, after students have worked with such methods and understand them well, they practice the ones they use routinely so they can use them efficiently.

Still later, when they are firmly grounded in understanding the operation and solving problems accurately, efficiently, and flexibly, they study and compare algorithms, including the U.S. traditional algorithms for addition, subtraction, and multiplication, in order to learn about the mathematical relationships underlying them. In particular, students' algorithms for solving addition and multiplication problems are usually quite similar to the historically taught algorithms in the United States; with a solid foundation in more transparent methods, students are ready to learn how the shorthand notation of these algorithms relates to the procedures they are carrying out but notating differently.

Students can come to appreciate the economy and generality of these traditional algorithms, which are an important achievement of mathematics, and use them along with other algorithms and methods. These algorithms are part of the knowledge in the world that students encounter, in their families, and elsewhere. They should be familiar, not mysterious. As with other algorithms and methods they use, students should understand how these algorithms work and what the notation represents.

RESEARCH

Update: What Did They Learn?

A Longitudinal, Comparative, and Focused Study of Mathematics Curricula

Paul Kehle, Hobart and William Smith Colleges, Geneva, NY

TERC is nearing completion of an NSF-funded revision of its curriculum, *Investigations in Number, Data, and Space*. A research team based at Indiana University, consisting of Diana Lambdin (Principal Investigator), Paul Kehle (Project Director, Hobart and William Smith Colleges), N. Kathryn Essex, Kelly McCormick (University of Southern Maine), has just completed data collection for a longitudinal, matched-comparison study of the prepublication version of the revised *Investigations* curriculum.

Approximately 1,800 students in three different geographical U.S. areas representing a range of socioeconomic and cultural characteristics were followed for three consecutive years. Students were in one of two cohorts: a Grade 1 cohort followed through Grades 1, 2, and 3, or a Grade 3 cohort followed through Grades 3, 4, and 5. Each child participating in the study completed a series of assessments. In addition to these results, we collected each child's results on any locally administered standardized tests. Finally, we also collected curriculum-implementation data to gauge the extent to which the students experienced their curriculum as intended by its authors.

The Indiana University assessments, designed to be curriculum-neutral, focus on the areas of number sense and operation, and early algebraic thinking. This selective focus allowed us to gain more detail about student achievement in these vital areas than a more comprehensive assessment and reduced the amount of testing time required of the students.

Because of the longitudinal design of the study, meaningful findings will emerge only after all the data have been analyzed. Furthermore, the final results will depend heavily on the final study attrition figures. As children moved in and out of the study sites, the number of participants decreased, and this is especially true in low-SES profile districts. Although the results of this study are primarily intended to help TERC in its curriculum development work, we hope to have meaningful results to share with educators in spring 2007.

CURRICULUM

CURRICULUM

Unit Descriptions

KINDERGARTEN

Classroom Routines and Materials

This unit introduces the processes, structures, and materials that are important features of the kindergarten math curriculum. It also introduces routines, common to many kindergarten classrooms, that students will encounter regularly throughout the year. These routines include taking attendance, using the calendar to count and to keep track of time and events, counting sets of objects, and collecting and discussing data about the class. They offer reinforcement of number concepts that are central to the kindergarten curriculum.

Counting and Comparing: *The Number System*

Students explore numbers through a variety of counting activities. They build knowledge of the counting sequence, use numerals to represent quantities, represent equivalent amounts, and develop skills for accurate counting. They also begin to compare quantities. As an introduction to linear measurement, students measure and compare the lengths of objects using direct comparison.

What Comes Next? *Patterns and Functions*

In this unit, students investigate what makes a repeating pattern. They focus on attributes of objects and think about which attributes (i.e., size, color, shape, orientation) are important in the patterns they are making. Students work with simple and complex repeating patterns. They have many opportunities to copy, create, and extend repeating patterns using a variety of materials and common objects. They use patterns to determine what comes next and focus on the part, or unit, of a pattern that repeats.

Measuring and Counting: *Measurement and the Number System*

Students gain a deeper understanding of numbers and number relationships as they engage in activities in which they count, combine, and compare amounts. They develop visual images of numbers and solve problems in which they find different combinations of the same number. Students are introduced to addition and subtraction situations through story problem contexts. Work with linear measurement continues as students use nonstandard units to measure the length of objects and paths.

Make a Shape, Build a Block: *2D and 3D Geometry*

Students explore geometry using a variety of materials, including Geoblocks, pattern blocks, interlocking cubes and geoboards. They describe, sort, and compose and decompose two- and three-dimensional shapes. They think about shapes in their environment and match two-dimensional shapes to three-dimensional objects.

The *Shapes* software is introduced as a tool for extending and deepening this work. This tool is designed for K–2 students to explore how different shapes go together, experiment with different sorts of geometric transformations (rotations, translation, reflection), explore patterning, and investigate symmetry.

How Many Do You Have? *Addition and the Number System*

Students continue to work with counting and number composition as they count sets of objects and find multiple combinations of the same number as they decompose numbers to 10. They use numbers and notation to describe arrangements of tiles and number combinations. Students continue to develop an understanding

of the operations of addition and subtraction as they act out, model, solve story problems, and play games that involve combining or separating small amounts.

Sorting and Surveys: *Data Analysis*

This unit develops ideas about sorting and classifying, counting, representing, conducting a data investigation, and using data to solve a problem. In this unit, students sort objects according to common attributes, as well as sort data about their class. They collect, record, and represent categorical and numerical data about their class, and they carry out their own data investigation by collecting responses to their own survey questions.

GRADE 1 UNIT SUMMARIES

How Many of Each? *Addition and the Number System*

In this first number unit in Grade 1, students count, order, compare quantities, and work with the operation of addition. Students also work on finding addition combinations up to 10 and learn to make sense of and solve story problems. They are introduced to several of the classroom routines that practice and reinforce work with counting, developing visual images of number, collecting data, and working with concepts of time (calendar/clock).

Making Shapes and Designing Quilts: *2D Geometry*

This geometry unit focuses on two-dimensional shapes and the relationships between them. Students observe, describe, compare, classify, represent, and compose and decompose 2-D shapes. Students learn to use geometric language to describe and identify important features of familiar 2-D shapes. As they sort and describe groups of shapes, they begin to distinguish specific attributes of triangles and quadrilaterals. As a final project, students create paper quilts by repeating combinations of triangles and squares.

The *Shapes* software is introduced as a tool for extending and deepening this work. This tool is designed for K–2 students to explore how different shapes go together, experiment with different sorts of geometric transformations (rotations, translation, reflection), explore patterning, and investigate symmetry.

Solving Story Problems: *Addition and Subtraction*

This unit focuses on counting to higher numbers (forward and back, counting sets of objects, comparing larger quantities, composing and decomposing numbers, and finding all the two-addend combinations of a number). Students revisit familiar addition activities with variations that encourage counting on, and they are introduced to the operation of subtraction. Through games and story problems, students' work focuses on developing an understanding of addition and subtraction, using numbers and notation to represent these operations, and developing strategies for solving addition and subtraction problems.

What Would You Rather Be? *Data Analysis*

Students pose questions and collect and sort information about data. They make representations of their findings and share them with others. Students sort a group of objects according to a given attribute.

Fish Lengths and Animal Jumps: *Measurement*

This unit focuses on developing the ideas about linear measurement, which include understanding what length is and developing a foundation of skills for accurate linear measurement using nonstandard and standard units. As students measure with a variety of units, they investigate the idea that different-sized units result in different measurements. Using a real-world context, students measure with inch tiles and grapple with the idea of partial units and “at least as long as,” ideas that are important in both measurement and number and operations. They also solve story problems, which involve comparing length.

Number Games and Crayon Puzzles: *Addition, Subtraction, and the Number System*

Students work on composing numbers with two and three addends, and in a variety of contexts, they work with combinations that equal 10 and explore relationships among those combinations. The addition and subtraction work of this unit continues to focus on making sense of the operations of addition and subtraction, practicing adding and subtracting single-digit numbers, and solving addition and subtraction story problems. There is a focus on naming and comparing different strategies used for solving problems such as counting all, counting on or back, and using known-number combinations. Students also discuss how different tools such as objects, the number line, and 100 chart can be used to model and solve problems.

Color, Shape, and Number Patterns: *Patterns and Functions*

Students make, describe, and extend repeating patterns. As they analyze the regularities of these patterns and identify the unit of the pattern that repeats, they build an understanding of what makes patterns predictable. They use this information to determine what comes next or what comes several steps ahead in a repeating pattern. Students also work on constructing, describing, and extending number sequences with a constant increase generated by various problem contexts.

Twos, Fives, and Tens: *Addition, Subtraction, and the Number System*

Students revisit the number sequence as they count and write numbers to 100 and beyond. Students work on achieving fluency with the two-addend combinations of ten, they are introduced to ideas about equivalence ($8 + 5 = 10 + 3$), and they engage in activities that highlight the importance of ten in our Base-10 number system. As students work with contexts that provide opportunities to count by groups of 2s, 5s, and 10s, they think about ways to organize objects so that they are easier to count and combine, and they begin to make sense of what it means to count by equal groups.

Blocks and Boxes: *3D Geometry*

This second geometry unit in first grade focuses on three-dimensional shapes and the relationships between them. Students observe, describe, compare, classify, represent, and build with 3D shapes. They develop vocabulary for naming and describing 3D shapes and explore the relationship between 2D and 3D shapes. In this unit, students focus on the attributes of rectangular prisms. As a final project, they use 3D shapes to construct a town and work with directions and paths as they plan routes through the town.

GRADE 2 UNIT SUMMARIES

Counting, Coins, and Combinations: Addition, Subtraction, and the Number System

This unit focuses on counting and comparing quantities, composing and decomposing numbers, and understanding the operations of addition and subtraction. Students develop strategies for comparing, combining, and doubling quantities, as well as taking one quantity away. They also achieve fluency with three sets of addition combinations (10s, + 1, + 2). During this first unit of the year, students are introduced to several year-long classroom routines that offer regular practice with composing and decomposing numbers; developing visual images of quantities; counting, collecting, and analyzing data; and telling time.

Shapes, Blocks, and Symmetry: 2D and 3D Geometry

Students identify two- and three-dimensional shapes, focus on the properties of rectangles and rectangular prisms, and identify and create symmetrical designs. Students also achieve fluency with the doubles addition combinations.

The *Shapes* software is introduced as a tool for extending and deepening this work. This tool is designed for K–2 students to explore how different shapes go together, experiment with different sorts of geometric transformations (rotations, translation, reflection), explore patterning, and investigate symmetry.

Stickers, Number Strings, and Story Problems: Addition, Subtraction, and the Number System

In this second number unit, students solve problems with multiple addends and consider whether order matters in addition. For example, does $7 + 4 + 3 + 6 = 7 + 3 + 4 + 6$? Students revisit addition and subtraction story problems, investigate even and odd numbers, and begin to make sense of counting by groups and place value (tens and ones). Work on addition combinations continues as students achieve fluency with the Near Doubles.

Pockets, Teeth, and Favorite Things: Data Analysis

Students engage in all the phases of data analysis as they pose questions, collect and sort information, and make representations of data as a way of sharing their findings with others. They work with Venn diagrams and line plots, and they read and interpret a variety of representations of numerical and categorical data. Students are also assessed on fluency with the +10 addition combinations.

How Many Floors? How Many Rooms? Patterns and Functions

In this unit, students describe and represent ratios, use tables to represent and predict change, and work with numeric sequences as they construct and describe patterns. Students extend repeating patterns and determine which element of the pattern will be in a particular position.

How Many Tens? How Many Ones? Addition, Subtraction, and the Number System

Students continue to build their understanding of place value (ones, tens, hundreds) as they compose and decompose numbers into tens and ones and work with contexts and models for the Base-10 number system. Students apply their work with place value as they play games that involve composing and decomposing 100 and solve addition and subtraction problems to 100. There is continuing work on developing coin equivalencies and combinations, developing visual images of numbers, and telling time.

Parts of a Whole, Parts of a Group: Fractions

Students investigate what fractions are and the many ways they can be represented and used. They identify fractions of a single object ($\frac{1}{2}$ of a square, $\frac{1}{4}$ of a rectangle, etc.) as well as find fractions of a set ($\frac{1}{2}$ of 12). Students begin to learn how fractions are expressed in words and represented using fraction notation.

Partners, Teams, and Paper Clips: Addition and Subtraction

This final number unit of Grade 2 is a culmination of the number and operations work students have done in Grade 2. Students refine their strategies for adding and subtracting numbers as they work toward developing fluency with addition and subtraction of two-digit numbers up to 100. They investigate and make generalizations about what happens when you add even and odd numbers. They learn the remaining single-digit addition combinations, achieving fluency with all single-digit addition combinations. Work continues with telling time, place value, and coin combinations.

Measuring Length and Time: Measurement

Students investigate linear measurement as it applies to length and distance. They work with a variety of linear units, including standard units of inches, feet, yards, centimeters, and meters. Students build on their work with telling time as they measure, record, and calculate duration of events using timelines and schedules.

GRADE 3 UNIT SUMMARIES**Trading Stickers, Combining Coins: Addition, Subtraction, and the Number System**

In this first of three addition, subtraction, and number system units, students solve addition problems with two- and small three-digit numbers, solve subtraction problems involving two-digit numbers, find combinations of numbers that add to 100, and work with coins and coin values. Their understanding of place value develops as they add and subtract 10s to and from three-digit numbers, break three-digit numbers into hundreds, tens, and ones in different ways, and solve story problems involving hundreds, tens, and ones.

Surveys and Line Plots: Data Analysis

In this unit, students collect, represent, describe, categorize, and interpret both categorical and numerical data. They begin the important work of seeing a data set as a whole as they design and carry out their own data investigations, create representations of the data collected, and compare and discuss these representations. Students draw conclusions about the data by identifying characteristics in their representations. (Where is most of the data clumped? Where is the mode?) Students' collection of numerical data includes measuring length in inches and feet. Students review the basic units of inches, feet, and yards and their relationships (e.g., one foot is equivalent to 12 inches, one yard is equivalent to 3 feet or 36 inches) by measuring lengths longer than one foot.

Collections and Travel Stories: Addition, Subtraction, and the Number System

Students practice and refine their strategies for solving addition problems with three-digit numbers to 400 and subtraction problems with two- and three-digit numbers to 300. In addition to solving removal problems, they expand their understanding of subtraction as they solve comparison problems and problems in which they find

the missing part of a whole. They increase their understanding of place-value as they extend their work into three-digit numbers up to 1,000 and study the structure of 1,000.

Perimeter, Angles, and Area: 2D Geometry and Measurement

This unit develops ideas about the attributes of 2D objects and how they are classified (the definition of a triangle, rectangle, and square), linear measurement (which includes perimeter), area, and the measurement of angles. Using the context of perimeter, students continue to develop their ability to use measurement tools as they work on accurate linear measurement techniques. Students learn to identify angles by their relationship to a right angle. (Is the angle greater than, less than, or equal to a 90-degree angle?) They develop an understanding of area as the amount of flat space an object covers and determine the area of 2D shapes in square units.

LogoPaths, a Logo programming environment designed for *Investigations* students in Grades 3–5 is introduced in this unit. It allows students to explore geometrical relationships, especially focusing on angle, length, and perimeter, patterns in sides and angles, and characteristics of specific shapes.

Equal Groups: Multiplication and Division

In this unit, students develop an understanding of multiplication as combining a number of equal groups and division as splitting a quantity into equal groups. This understanding is developed as students highlight multiples on 100 charts, describe patterns in sets of multiples, and compare sets of multiples to each other. Students represent multiplication and division situations with groups, rectangular arrays, and by writing multiplication and division story problems. In this unit, students achieve fluency with multiplication combinations with products to 50 and consider the relationship between multiplication and division (e.g., $6 \times 4 = 24$; $24 \div 6 = 4$).

Stories, Tables, and Graphs: Patterns, Functions, and Change

In this unit, students use tables and line graphs to show changes over time. These include representations of functions involving constant change that can be described by mathematical rules and functions involving variable changes that cannot, such as temperature over time. Students interpret graphs to describe and compare temperature trends. They also compare different situations of constant change. In addition, students construct, describe, and extend visual and number sequences.

Finding Fair Shares: Fractions and Decimals

Using a variety of contexts (rectangles representing “brownies,” pattern block “cookies,” and groups of objects), students develop their understanding of fractions as representing equal parts of a whole. They work with commonly used fractions and their equivalencies (e.g., $3/6$ and $2/4$ both equal one-half of the same whole) and use fractions and mixed numbers as they solve sharing problems and build wholes from fractional parts. They are introduced to decimal fractions in the context of money and gain familiarity with decimal equivalents for one-fourth and one-half.

How Many Hundreds? How Many Miles? *Addition and Subtraction*

In this final addition and subtraction unit in Grade 3, students further refine their addition strategies with problems involving any three-digit numbers as they identify and name these strategies. They continue to solve a variety of subtraction problems and examine their strategies for subtracting any two- and three-digit numbers in greater depth. Students extend their work with place value by estimating the sum of three-digit numbers (Will the sum be more or less than 400?) and adding and subtracting multiples of 10 and 100 to and from three-digit numbers. In this unit, students are assessed on fluency with subtraction facts.

Solids and Boxes: *3D Geometry and Measurement*

Students develop ideas about the attributes of 3D shapes and how these attributes determine classification as they sort and build common geometric solids. Throughout the unit, they move back and forth between 2D and 3D as they build and describe 2D representations of 3D objects and create 3D objects from their 2D representations. They begin to develop important ideas about the measurement of volume as they examine the structure of 2D box patterns and the number of cubes the 3D box will hold.

GRADE 4 UNIT SUMMARIES**Factors, Multiples, and Arrays: *Multiplication***

In this first unit in the multiplication and division strand, students deepen their understanding of the operation of multiplication. Students use rectangular arrays to represent the relationship between factors and multiples, use what they know to solve problems that increase in size, and focus on solving problems efficiently. They continue to develop fluency with multiplication combinations (facts up to 12×12).

Describing the Shape of the Data: *Data Analysis and Probability*

This unit develops ideas about collecting, describing, and representing data. Students collect data through counting and measuring and use bar graphs and line plots to represent their data involving one group and compare data of two groups. They interpret the data and draw conclusions based on the data using terms such as mode, median, range, and outlier. Students begin their study of probability by placing events on a “likelihood line” that goes from impossible to certain and work with mixtures of colored cubes to describe the probability of different outcomes.

Multiple Towers and Division Stories: *Multiplication and Division*

Students develop strategies for solving multiplication problems with two-digit numbers and deepen their understanding of the operation of division by focusing on the relationship between multiplication and division. Using story contexts and multiple towers, students continue their investigation of the relationship between numbers and their factors. Students practice multiplying by 10 and multiples of 10, break problems into smaller parts that can be multiplied easily, and find the multiples of two-digit numbers. They gain fluency with all multiplication combinations to 12×12 . Students solve, represent, and discuss division story problems, including some that have a remainder.

Size, Shape, and Symmetry: 2D Geometry and Measurement

This first geometry and measurement unit focuses on classifying two-dimensional shapes, comparing the size of angles, and working with linear and area measurement. Students define and categorize polygons by identifying sets of shapes that have a common attribute and use 90 degrees as a reference for finding the measurement of other angles. They continue their measurement work from earlier grades by measuring distance and perimeter, using both U.S. and metric units and finding the area of polygons in square units.

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Landmarks and Large Numbers: Addition, Subtraction, and the Number System

Students extend their knowledge of the number system by examining the structure of 10,000 and practice and refine strategies for adding and subtracting whole numbers up to 10,000. They continue to study place value by adding and subtracting multiples of 10 and 100 to numbers in the thousands, and they consolidate their understanding of the operation of addition by studying a variety of addition strategies and algorithms, including the U.S. algorithm for addition. Students continue their study of subtraction by solving, representing, and discussing their strategies for a variety of subtraction problems.

Fraction Cards and Decimal Squares: Rational Numbers

Students develop ideas about fractions by identifying fractions of an area ($\frac{3}{4}$ of a rectangle), fractions of a group of objects ($\frac{3}{4}$ of 24), and decimal fractions (.75). They compare fractions of different wholes ($\frac{1}{3}$ of a 6 x 4 rectangle and $\frac{1}{3}$ of a 10 x 10 rectangle), and combine fractions using models and reasoning. Students use 10 x 10 grids to represent, compare, and combine common decimals in the tenths and hundredths.

Moving Between Solids and Silhouettes: 3D Geometry and Measurement

In this second geometry and measurement unit, students examine the relationships between 3D solids and their 2D representations. They learn and use the mathematical terminology for these solids and their attributes. They translate between 3D shapes and their 2D representations as they build cube configurations from pictures and mental images and investigate silhouettes of solids from several different perspectives. Students build an understanding of measuring volume as they examine the structure of rectangular prisms and determine the number of cubes that fit inside given box patterns.

How Many Packages? How Many Groups? Multiplication and Division

In this last multiplication and division unit in Grade 4, students continue to develop efficient strategies for solving multiplication problems by breaking problems into smaller parts or changing one or both numbers to create an easier problem. Students also focus on recording their work with clear and concise notation. Students develop strategies for solving division problems (three-digit divided by two-digit), which involve making groups of the divisor. These problems are presented both in story contexts and numerically.

Penny Jars and Plant Growth: *Patterns, Functions, and Change*

Students explore situations in which two quantities change in relation to each other. They work with changes over time, such as increasing or decreasing speed or the growth of a plant, and situations of constant change, such as how the number of windows in a building depends on the height of the building if every floor has the same number of windows. Students create and interpret graphs and tables for these linear and nonlinear functions and connect these graphs to the situations they represent.

GRADE 5 UNIT SUMMARIES**Number Puzzles and Multiple Towers: *Multiplication and Division***

In the first of two units about multiplication and division, students continue to develop and practice efficient strategies to solve multiplication problems both in and out of a context. Students refine and gain fluency in solving two-digit by two-digit multiplication problems, develop strategies for division problems with one- and two-digit divisors, and increase their knowledge of multiplication relationships by learning about prime factorization (e.g., $36 = 4 \times 9 = (2 \times 2) \times 9 = 2 \times 2 \times 3 \times 3$).

Prisms and Pyramids: *3D Geometry and Measurement*

Students investigate concepts of volume by finding the volume of prisms, pyramids, cylinders, and cones. They use patterns of open boxes and build prisms from cubes to develop a strategy for finding the volume of any rectangular prism. Using concrete materials, they also examine the 3-to-1 volume relationship between related (having the same base and height) prisms and pyramids, and related cylinders and cones. Geometry work includes naming geometric solids and their attributes.

Thousands of Miles, Thousands of Seats: *Addition, Subtraction, and the Number System*

Students study place value in large numbers by building a 10,000 chart and by adding multiples of 10 to and subtracting multiples of 10 from four- and five-digit numbers. Students finalize their study of subtraction by refining and gaining fluency in solving subtraction problems, including a study of the U.S. algorithm for subtraction. Using a context of the capacities of stadiums and arenas, they solve addition and subtraction problems involving four- and five-digit numbers. Students also demonstrate fluency with the division facts up to $144 \div 12$.

What's That Portion? *Fractions and Percents*

Students study the relationship among fractions and between fractions and percents and use this knowledge to find equivalent fractions, order fractions, and add commonly used fractions. They use a variety of contexts and models, including area, number lines, and rotation, to further understand the meaning of fractions and model their strategies. This is the first of two units in Grade 5 about rational numbers.

Measuring Polygons: 2D Geometry and Measurement

Students create polygons using “power polygon” pieces and discuss, apply, and evaluate definitions of these polygons. They focus on properties of quadrilaterals and also study similarity of 2D shapes. Measurement work includes finding measures of angles using known angles and finding perimeter and area of rectangles.

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Decimals on Grids and Number Lines: Decimals, Fractions, and Percents

In this unit, students focus primarily on decimals and how the number system extends to numbers less than one. Students use their understanding of fractions and percents and their relationship to decimals to solve computation problems involving decimals. Students use contexts (time and precipitation) and models (area and number lines) to order and add decimals. This is the second of two rational number units in Grade 5.

How Many People, How Many Teams: Multiplication and Division

In the second unit on multiplication and division, students find and study equivalent expressions for multiplication and division problems (e.g., $112 \div 8 = 28 \div 2$). Students practice solving larger multiplication problems (3 digit x 2 digit) accurately and efficiently and study the U.S. algorithm for multiplication. Students gain fluency solving division problems with one-, two-, and three-digit divisors.

Growth Patterns: Patterns, Functions, and Change

Students investigate situations in which two quantities change in relation to each other. Students describe data about functional relationships, develop an overall sense of change from a graph, and understand how the changes and totals are related. They also compare two linear functions with different rates of change.

How Long Can You Stand on One Foot? Data Analysis and Probability

Students describe major features of a set of data, represented in a line plot or bar graph, and quantify the description by using medians or fractional parts of the data. Students draw conclusions about how two groups compare based on summarizing the data for each group. They conduct their own data experiment. Students also look at the probability of various events. They express the probabilities of the possible outcomes (e.g., landing on the green part of the spinner, landing on the white part of the spinner) by using numbers from 0 to 1. Then they conduct experiments to see what actually occurs. Students also consider the notion of fairness in the context of probability by playing fair and unfair games, that is, games in which players do or do not have equal chances of winning.

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